Answer Set Solving in Practice

Torsten Schaub

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Outline

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Goal: Declarative problem solving

- *"What is the problem?"*

  instead of
- *"How to solve the problem?"*

$$\begin{tikzcd}
\text{Problem} \arrow[d, "\text{Modeling}"'] & \text{Solution} \\
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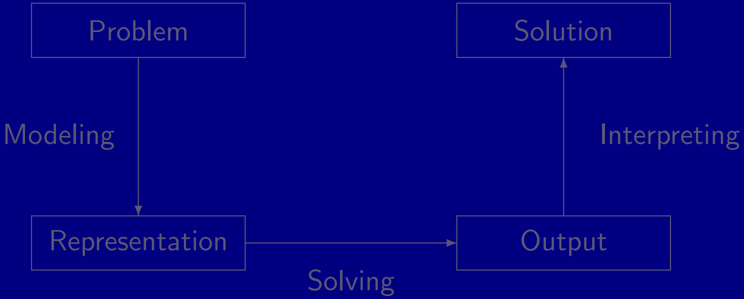

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Answer Set Programming (ASP) *in a Nutshell*

- ASP is an approach to declarative problem solving, combining
  - a rich yet simple modeling language
  - with high-performance solving capacities

  tailored to Knowledge Representation and Reasoning
- ASP allows for solving all search problems in NP (and NP^{NP}) in a uniform way (being more compact than SAT)
- The versatility of ASP is reflected by the ASP solver `clasp`, winning first places at ASP, CASC, MISC, PB, and SAT
  - `http://potassco.sourceforge.net`
- ASP embraces many emerging application areas, eg.
  - RoboCup@Home at USTC, Beijing
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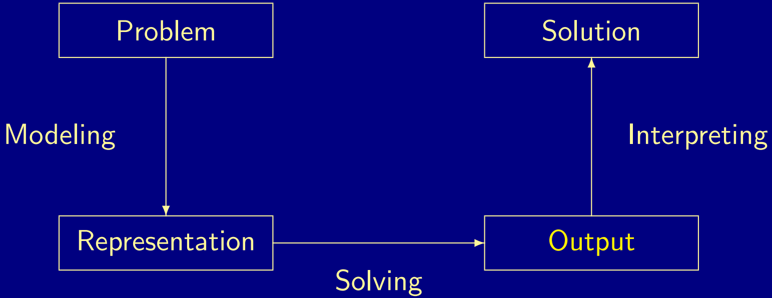

Declarative model-based problem solving

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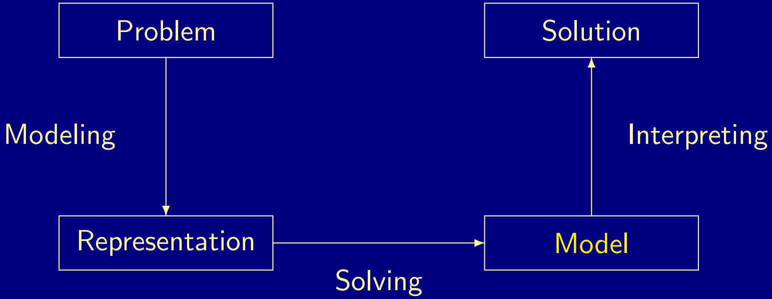

Basic idea

Consider the logical formula Φ and its three (classical) models:

$$\{p, q\}, \{q, r\}, \text{ and } \{p, q, r\}$$

$$\Phi \quad \boxed{q \;\wedge\; (q \wedge \neg r \rightarrow p)}$$

Formula Φ has one stable model, often called answer set:

$$\{p, q\}$$

$$P_\Phi \quad \boxed{\begin{array}{lll} q & \leftarrow & \\ p & \leftarrow & q, \sim r \end{array}}$$

Informally, a set X of atoms is an stable model of a logic program P

if X is a (classical) model of P and

if all atoms in X are justified by some rule in P

(rooted in intuitionistic logics HT (Heyting, 1930) and G3 (Gödel, 1932))

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$$\boxed{\begin{array}{ccc} p & \mapsto & 1 \\ q & \mapsto & 1 \\ r & \mapsto & 0 \end{array}}$$

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ASP versus SAT

ASP	SAT
Model generation	
Bottom-up	
Constructive Logic	Classical Logic
Closed (and open) world reasoning	Open world reasoning
Modeling language	—
Complex reasoning modes	Satisfiability testing
Satisfiability	Satisfiability
Enumeration/Projection	—
Optimization	—
Intersection/Union	—
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Formal Definition

- Syntax
  - A rule, r, is an expression of the form

    $$a \leftarrow b_1, \dots, b_m, \sim c_1, \dots, \sim c_n,$$

    where $0 \leq m, n$ and each a, b_i, c_j is an atom
  - A logic program is a finite set of rules

- Semantics

  The reduct, P^X, of a program P relative to a set X of atoms is defined by

  $$P^X = \{\, a \leftarrow b_1, \dots, b_m \mid r \in P \text{ and } \{c_1, \dots, c_n\} \cap X = \emptyset \}$$

  The $\subseteq$–smallest model of P^X is denoted by $Cn(P^X)$
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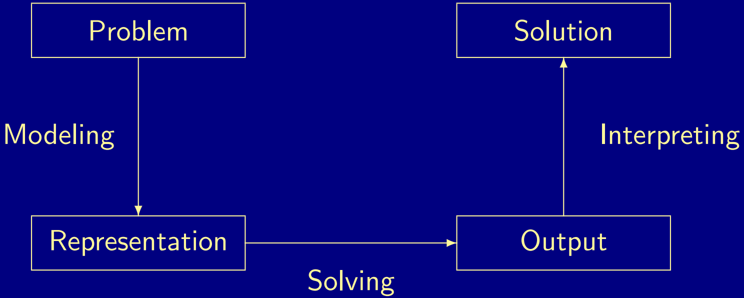

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Language Constructs

- Variables (over the Herbrand Universe)
  - `p(X) :- q(X)` over constants $\{a,b,c\}$ stands for
    `p(a) :- q(a), p(b) :- q(b), p(c) :- q(c)`
- Conditional Literals
  - `p :- q(X) : r(X)` given `r(a), r(b), r(c)` stands for
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- Disjunction
  - `p(X) ; q(X) :- r(X)`
- Integrity Constraints
  - `:- q(X), p(X)`
- Choice
  - `2 { p(X,Y) : q(X) } 7 :- r(Y)`
- Aggregates
  - `s(Y) :- r(Y), 2 #count { p(X,Y) : q(X) } 7`
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Reasoning Modes

- Satisfiability
- Enumeration†
- Projection†
- Intersection‡
- Union‡
- Optimization

† without solution recording

‡ without solution enumeration

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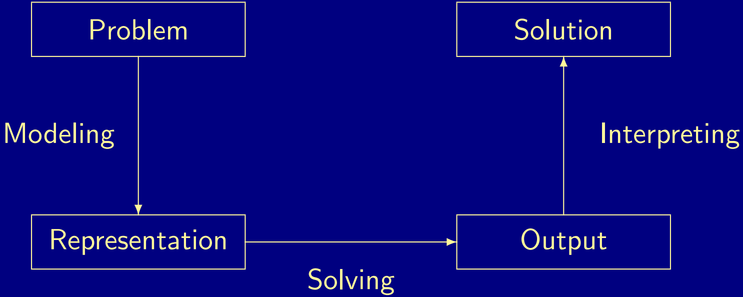
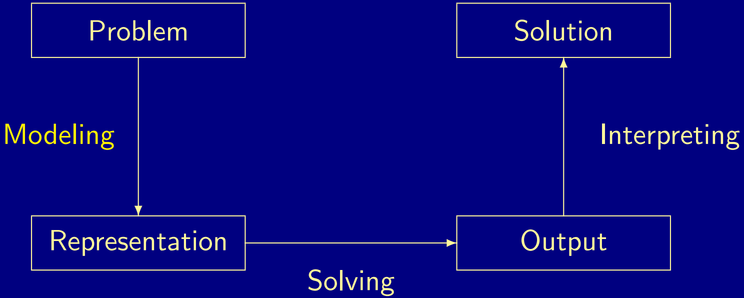
  instead of
- *"How to solve the problem?"*


$$\begin{tikzcd} \boxed{\text{Problem}} \arrow[d, "\text{Modeling}"'] & \boxed{\text{Solution}} \\ \boxed{\text{Representation}} \arrow[r, "\text{Solving}"'] & \boxed{\text{Output}} \arrow[u, "\text{Interpreting}"'] \end{tikzcd}$$

Declarative problem solving

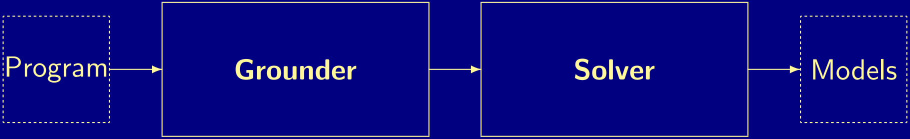
- *"What is the problem?"*

  instead of
- *"How to solve the problem?"*

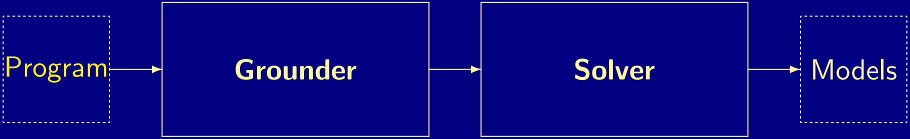


$$\begin{tikzcd}
\text{Problem} \arrow[d, "\text{Modeling}"'] & & \text{Solution} \\
\text{Representation} \arrow[rr, "\text{Solving}"'] & & \text{Output} \arrow[u, "\text{Interpreting}"']
\end{tikzcd}$$

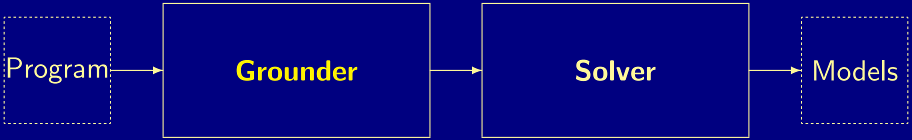
ASP Solving Process



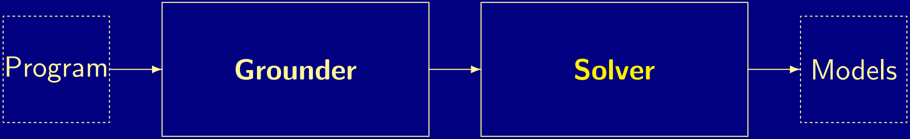
ASP Solving Process



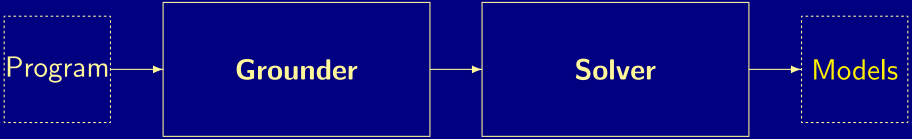
ASP Solving Process



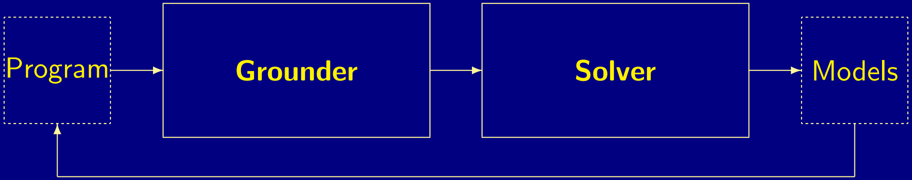
ASP Solving Process



ASP Solving Process

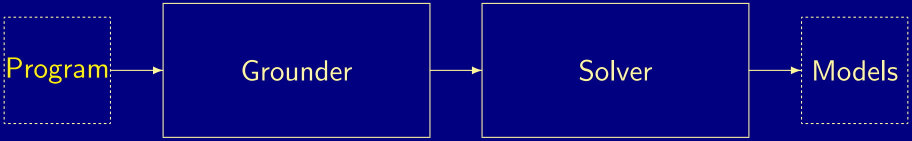


ASP Solving Process



Outline

ASP Solving Process



Graph Coloring

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).   col(b).   col(g).


1 { color(X,C) : col(C) } 1 :- node(X).


:- edge(X,Y), color(X,C), color(Y,C).
```

Graph Coloring

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).   col(b).   col(g).


1 { color(X,C) : col(C) } 1 :- node(X).


:- edge(X,Y), color(X,C), color(Y,C).
```

Graph Coloring

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).   col(b).   col(g).


1 { color(X,C) : col(C) } 1 :- node(X).


:- edge(X,Y), color(X,C), color(Y,C).
```

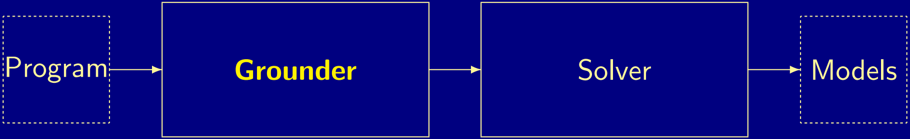
Graph Coloring

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).  col(b).  col(g).


1 { color(X,C) : col(C) } 1 :- node(X).


:- edge(X,Y), color(X,C), color(Y,C).
```

Graph Coloring

```
node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).   col(b).   col(g).


1 { color(X,C) : col(C) } 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
```

ASP Solving Process



Graph Coloring: Grounding

```
$ gringo -t color.lp
```

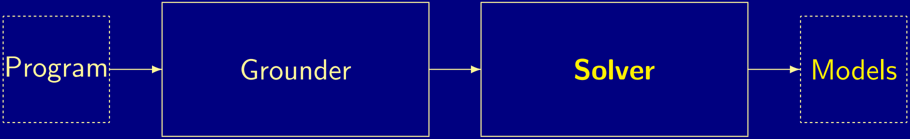
```
node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

edge(1,2).  edge(1,3).  edge(1,4).  edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).  edge(4,1).  edge(4,2).  edge(5,3).
edge(5,4).  edge(5,6).  edge(6,2).  edge(6,3).  edge(6,5).

col(r).  col(b).  col(g).

1 {color(1,r), color(1,b), color(1,g)} 1.
1 {color(2,r), color(2,b), color(2,g)} 1.
1 {color(3,r), color(3,b), color(3,g)} 1.
1 {color(4,r), color(4,b), color(4,g)} 1.
1 {color(5,r), color(5,b), color(5,g)} 1.
1 {color(6,r), color(6,b), color(6,g)} 1.

 :- color(1,r), color(2,r).  :- color(2,g), color(5,g).  ...  :- color(6,r), color(2,r).
 :- color(1,b), color(2,b).  :- color(2,r), color(6,r).       :- color(6,b), color(2,b).
 :- color(1,g), color(2,g).  :- color(2,b), color(6,b).       :- color(6,g), color(2,g).
 :- color(1,r), color(3,r).  :- color(2,g), color(6,g).       :- color(6,r), color(3,r).
 :- color(1,b), color(3,b).  :- color(3,r), color(1,r).       :- color(6,b), color(3,b).
 :- color(1,g), color(3,g).  :- color(3,b), color(1,b).       :- color(6,g), color(3,g).
 :- color(1,r), color(4,r).  :- color(3,g), color(1,g).       :- color(6,r), color(5,r).
 :- color(1,b), color(4,b).  :- color(3,r), color(4,r).       :- color(6,b), color(5,b).
 :- color(1,g), color(4,g).  :- color(3,b), color(4,b).       :- color(6,g), color(5,g).
 :- color(2,r), color(4,r).  :- color(3,g), color(4,g).
 :- color(2,b), color(4,b).  :- color(3,r), color(5,r).
 :- color(2,g), color(4,g).  :- color(3,b), color(5,b).
```

Graph Coloring: Grounding

```
$ gringo -t color.lp
```

```
node(1).  node(2).  node(3).  node(4).  node(5).  node(6).

edge(1,2).  edge(1,3).  edge(1,4).  edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).  edge(4,1).  edge(4,2).  edge(5,3).
edge(5,4).  edge(5,6).  edge(6,2).  edge(6,3).  edge(6,5).

col(r).  col(b).  col(g).

1 {color(1,r), color(1,b), color(1,g)} 1.
1 {color(2,r), color(2,b), color(2,g)} 1.
1 {color(3,r), color(3,b), color(3,g)} 1.
1 {color(4,r), color(4,b), color(4,g)} 1.
1 {color(5,r), color(5,b), color(5,g)} 1.
1 {color(6,r), color(6,b), color(6,g)} 1.

 :- color(1,r), color(2,r).  :- color(2,g), color(5,g).  ...  :- color(6,r), color(2,r).
 :- color(1,b), color(2,b).  :- color(2,r), color(6,r).       :- color(6,b), color(2,b).
 :- color(1,g), color(2,g).  :- color(2,b), color(6,b).       :- color(6,g), color(2,g).
 :- color(1,r), color(3,r).  :- color(2,g), color(6,g).       :- color(6,r), color(3,r).
 :- color(1,b), color(3,b).  :- color(3,r), color(1,r).       :- color(6,b), color(3,b).
 :- color(1,g), color(3,g).  :- color(3,b), color(1,b).       :- color(6,g), color(3,g).
 :- color(1,r), color(4,r).  :- color(3,g), color(1,g).       :- color(6,r), color(5,r).
 :- color(1,b), color(4,b).  :- color(3,r), color(4,r).       :- color(6,b), color(5,b).
 :- color(1,g), color(4,g).  :- color(3,b), color(4,b).       :- color(6,g), color(5,g).
 :- color(2,r), color(4,r).  :- color(3,g), color(4,g).
 :- color(2,b), color(4,b).  :- color(3,r), color(5,r).
 :- color(2,g), color(4,g).  :- color(3,b), color(5,b).
```

ASP Solving Process



Graph Coloring: Solving

```
$ gringo color.lp | clasp 0
```

```
clasp version 2.1.0
Reading from stdin
Solving...
Answer: 1
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,g) color(4,b) color(3,r) color(2,r) color(1,g)
Answer: 2
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,g) color(4,r) color(3,b) color(2,b) color(1,g)
Answer: 3
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,b) color(4,g) color(3,r) color(2,r) color(1,b)
Answer: 4
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,b) color(4,r) color(3,g) color(2,g) color(1,b)
Answer: 5
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,r) color(4,g) color(3,b) color(2,b) color(1,r)
Answer: 6
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,r) color(4,b) color(3,g) color(2,g) color(1,r)
SATISFIABLE

Models      : 6
Time        : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.000s
```

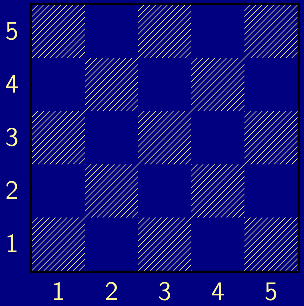
Graph Coloring: Solving

```
$ gringo color.lp | clasp 0
```

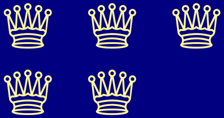
```
clasp version 2.1.0
Reading from stdin
Solving...
Answer: 1
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,g) color(4,b) color(3,r) color(2,r) color(1,g)
Answer: 2
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,g) color(4,r) color(3,b) color(2,b) color(1,g)
Answer: 3
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,b) color(4,g) color(3,r) color(2,r) color(1,b)
Answer: 4
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,b) color(4,r) color(3,g) color(2,g) color(1,b)
Answer: 5
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,r) color(4,g) color(3,b) color(2,b) color(1,r)
Answer: 6
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,r) color(4,b) color(3,g) color(2,g) color(1,r)
SATISFIABLE

Models      : 6
Time        : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time    : 0.000s
```

Outline

The n-Queens Problem



- Place n queens on an $n \times n$ chess board
- Queens must not attack one another



Defining the Field

queens.lp

```
row(1..n).
col(1..n).
```

- Create file queens.lp
- Define the field
  - *n* rows
  - *n* columns

Defining the Field

Running . . .

```
$ clingo queens.lp -c n=5
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
SATISFIABLE

Models      : 1
Time        : 0.000
  Prepare   : 0.000
  Prepro.   : 0.000
  Solving   : 0.000
```

Placing some Queens

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
```

- Guess a solution candidate
  by placing some queens on the board

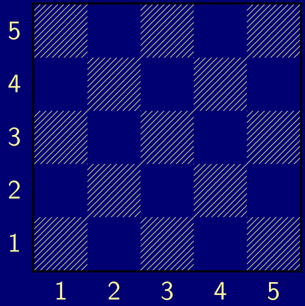
Placing some Queens

Running . . .

```
$ clingo queens.lp -c n=5 3
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5)
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) queen(1,1)
Answer: 3
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) queen(2,1)
SATISFIABLE

Models       : 3+
```

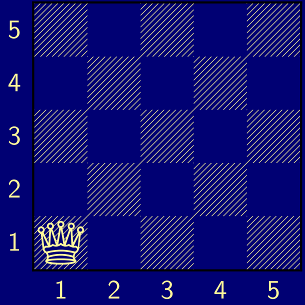
Placing some Queens: Answer 1

Answer 1



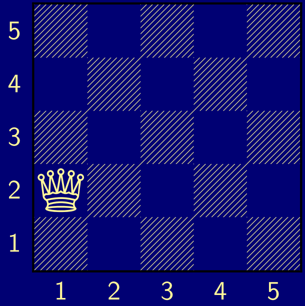
Placing some Queens: Answer 2

Answer 2



Placing some Queens: Answer 3

Answer 3



Placing *n* Queens

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not n { queen(I,J) } n.
```

- Place exactly *n* queens on the board

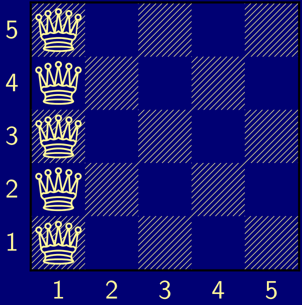
Placing *n* Queens

Running . . .

```
$ clingo queens.lp -c n=5 2
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(5,1) queen(4,1) queen(3,1) \
queen(2,1) queen(1,1)
Answer: 2
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(1,2) queen(4,1) queen(3,1) \
queen(2,1) queen(1,1)
...
```

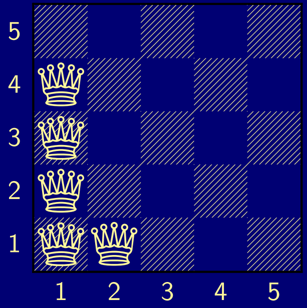
Placing *n* Queens: Answer 1

Answer 1



Placing *n* Queens: Answer 2

Answer 2



Horizontal and vertical Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not n { queen(I,J) } n.
:- queen(I,J), queen(I,JJ), J != JJ.
:- queen(I,J), queen(II,J), I != II.
```

- Forbid horizontal attacks
- Forbid vertical attacks

Horizontal and vertical Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not n { queen(I,J) } n.
:- queen(I,J), queen(I,JJ), J != JJ.
:- queen(I,J), queen(II,J), I != II.
```

- Forbid horizontal attacks
- Forbid vertical attacks

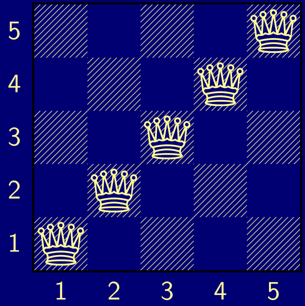
Horizontal and vertical Attack

Running . . .

```
$ clingo queens.lp -c n=5
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(5,5) queen(4,4) queen(3,3) \
queen(2,2) queen(1,1)
...
```

Horizontal and vertical Attack: Answer 1

Answer 1



Diagonal Attack

queens.lp

```
row(1..n).
col(1..n).
{ queen(I,J) : row(I) : col(J) }.
:- not n { queen(I,J) } n.
:- queen(I,J), queen(I,JJ), J != JJ.
:- queen(I,J), queen(II,J), I != II.
:- queen(I,J), queen(II,JJ), (I,J) != (II,JJ), I-J == II-JJ.
:- queen(I,J), queen(II,JJ), (I,J) != (II,JJ), I+J == II+JJ.
```

- Forbid diagonal attacks

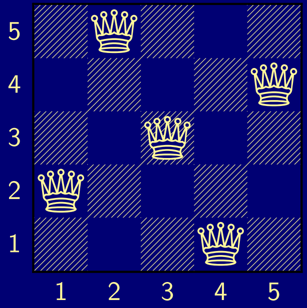
Diagonal Attack

Running . . .

```
$ clingo queens.lp -c n=5
Answer: 1
row(1) row(2) row(3) row(4) row(5) \
col(1) col(2) col(3) col(4) col(5) \
queen(4,5) queen(1,4) queen(3,3) \
queen(5,2) queen(2,1)
SATISFIABLE

Models      : 1+
Time        : 0.000
  Prepare   : 0.000
  Prepro.   : 0.000
  Solving   : 0.000
```

Diagonal Attack: Answer 1

Answer 1



Optimizing

queens-opt.lp

```
1 { queen(I,1..n) } 1 :- I = 1..n.
1 { queen(1..n,J) } 1 :- J = 1..n.
 :- 2 { queen(D-J,J) }, D =   2..2*n.
 :- 2 { queen(D+J,J) }, D = 1-n..n-1.
```

- Encoding can be optimized
- Much faster to solve

Outline

Traveling Salesperson

```
node(1..6).

edge(1,2;3;4).  edge(2,4;5;6).  edge(3,1;4;5).
edge(4,1;2).    edge(5,3;4;6).  edge(6,2;3;5).

cost(1,2,2).  cost(1,3,3).  cost(1,4,1).
cost(2,4,2).  cost(2,5,2).  cost(2,6,4).
cost(3,1,3).  cost(3,4,2).  cost(3,5,2).
cost(4,1,1).  cost(4,2,2).
cost(5,3,2).  cost(5,4,2).  cost(5,6,1).
cost(6,2,4).  cost(6,3,3).  cost(6,5,1).
```

Traveling Salesperson

```
node(1..6).

edge(1,2;3;4).  edge(2,4;5;6).  edge(3,1;4;5).
edge(4,1;2).    edge(5,3;4;6).  edge(6,2;3;5).

cost(1,2,2).  cost(1,3,3).  cost(1,4,1).
cost(2,4,2).  cost(2,5,2).  cost(2,6,4).
cost(3,1,3).  cost(3,4,2).  cost(3,5,2).
cost(4,1,1).  cost(4,2,2).
cost(5,3,2).  cost(5,4,2).  cost(5,6,1).
cost(6,2,4).  cost(6,3,3).  cost(6,5,1).
```

Traveling Salesperson

```
node(1..6).

edge(1,2;3;4).  edge(2,4;5;6).  edge(3,1;4;5).
edge(4,1;2).    edge(5,3;4;6).  edge(6,2;3;5).

cost(1,2,2).  cost(1,3,3).  cost(1,4,1).
cost(2,4,2).  cost(2,5,2).  cost(2,6,4).
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Traveling Salesperson

```
1 { cycle(X,Y) : edge(X,Y) } 1 :- node(X).
1 { cycle(X,Y) : edge(X,Y) } 1 :- node(Y).

reached(Y) :- cycle(1,Y).
reached(Y) :- cycle(X,Y), reached(X).

:- node(Y), not reached(Y).

#minimize { cycle(X,Y) : cost(X,Y,C) = C }.
```

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What is ASP good for?

- Combinatorial search problems in the realm of P, NP, and NP^{NP} (some with substantial amount of data), like
  - For instance, auctions, bio-informatics, computer-aided verification, configuration, constraint satisfaction, diagnosis, information integration, planning and scheduling, security analysis, semantic web, wire-routing, zoology and linguistics, and many more
- My favorite: Using ASP as a basis for a decision support system for NASA's space shuttle (Gelfond et al., Texas Tech)
- Our own applications:
  - Automatic synthesis of multiprocessor systems
  - Inconsistency detection, diagnosis, repair, and prediction in large biological networks
  - Home monitoring for risk prevention in ambient assisted living
  - General game playing

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What does ASP offer?

- Integration of KR, DB, and SAT techniques
- Succinct, elaboration-tolerant problem representations
    - Rapid application development tool
- Easy handling of dynamic, knowledge intensive applications
    - including: data, frame axioms, exceptions, defaults, closures, etc.

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$$\textbf{ASP} = \textbf{DB}+\textbf{LP}+\textbf{KR}+\textbf{SAT}$$

Outline

Declarative problem solving

- *"What is the problem?"*

  instead of
- *"How to solve the problem?"*

$$\begin{tikzcd}
\boxed{\text{Problem}} \arrow[d, "\text{Modeling}"'] & & \boxed{\text{Solution}} \\
\boxed{\text{Representation}} \arrow[rr, "\text{Solving}"'] & & \boxed{\text{Output}} \arrow[u, "\text{Interpreting}"']
\end{tikzcd}$$

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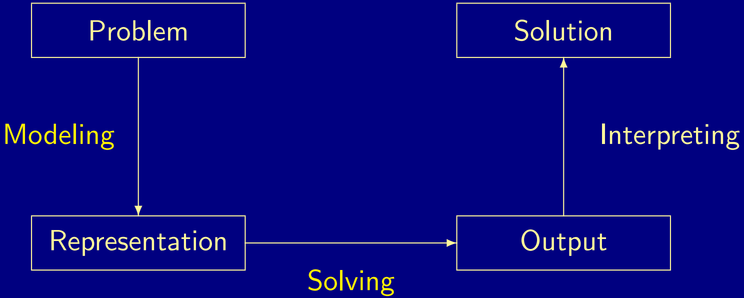
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\end{tikzcd}$$

Grounding by example

easy.lp

```
{ p(1..3) }.
:- { p(X) } 2.
q(X) :- p(X), p(X+1), X>1.
```

gringo --text easy.lp

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#count{ p(1), p(2), p(3) }.
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Solving by example

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gringo easy.lp | clasp 0

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clasp version 2.0.0
Reading from stdin
Solving...
Answer: 1
p(1) p(2) p(3) q(2)
SATISFIABLE

Models      : 1
Time        : 0.000s
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Reifying by example

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gringo --reify easy.lp

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rule(pos(false),pos(conjunction(1))).
set(2,pos(atom(p(2)))).
set(2,pos(atom(p(3)))).
rule(pos(atom(q(2))),pos(conjunction(2))).
```

Reifying by example

```
gringo --text easy.lp
```

```
    #count{ p(1), p(2), p(3) }.
    :- #count{ p(3), p(2), p(1) } 2.
    q(2) :- p(2), p(3).
```

```
gringo --reify easy.lp
```

```
    wlist(0,0,pos(atom(p(1))),1).
    wlist(0,1,pos(atom(p(2))),1).
    wlist(0,2,pos(atom(p(3))),1).
    rule(pos(sum(0,0,3)),pos(conjunction(0))).
    set(1,pos(sum(0,0,2))).
    rule(pos(false),pos(conjunction(1))).
    set(2,pos(atom(p(2)))).
    set(2,pos(atom(p(3)))).
    rule(pos(atom(q(2))),pos(conjunction(2))).
```

Reifying by example

```
gringo --text easy.lp
```

```
#count{ p(1), p(2), p(3) }.
:- #count{ p(3), p(2), p(1) } 2.
q(2) :- p(2), p(3).
```

```
gringo --reify easy.lp
```

```
wlist(0,0,pos(atom(p(1))),1).
wlist(0,1,pos(atom(p(2))),1).
wlist(0,2,pos(atom(p(3))),1).
rule(pos(sum(0,0,3)),pos(conjunction(0))).
set(1,pos(sum(0,0,2))).
rule(pos(false),pos(conjunction(1))).
set(2,pos(atom(p(2)))).
set(2,pos(atom(p(3)))).
rule(pos(atom(q(2))),pos(conjunction(2))).
```

Basic meta-encoding
`meta.lp`

```
litb(B) :- rule(_,B).
litb(E) :- litb(pos(conjunction(S))), set(S,E).
litb(E) :- eleb(sum(_,S,_)), wlist(S,_,E,_).

eleb(P) :- litb(pos(P)).
eleb(N) :- litb(neg(N)).

elem(E) :- eleb(E).
elem(E) :- rule(pos(E),_).
elem(P) :- rule(pos(sum(_,S,_)),_), wlist(S,_,pos(P),_).
elem(N) :- rule(pos(sum(_,S,_)),_), wlist(S,_,neg(N),_).

hold(conjunction(S)) :- eleb(conjunction(S)),
                            hold(P)     : set(S,pos(P)),
                        not hold(N)     : set(S,neg(N)).

hold(sum(L,S,U))     :- eleb(sum(L,S,U)),
                        L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
                                 not hold(N) = W : wlist(S,Q,neg(N),W) ] U.

hold(atom(A))        :- rule(pos(atom(A)),   pos(B)), hold(B).

 L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
          not hold(N) = W : wlist(S,Q,neg(N),W) ] U
                     :- rule(pos(sum(L,S,U)),pos(B)), hold(B).

                     :- rule(pos(false),     pos(B)), hold(B).

#hide. #show hold(atom(A)).
```

Basic meta-encoding `meta.lp`

```
litb(B) :- rule(_,B).
litb(E) :- litb(pos(conjunction(S))), set(S,E).
litb(E) :- eleb(sum(_,S,_)), wlist(S,_,E,_).

eleb(P) :- litb(pos(P)).
eleb(N) :- litb(neg(N)).

elem(E) :- eleb(E).
elem(E) :- rule(pos(E),_).
elem(P) :- rule(pos(sum(_,S,_)),_), wlist(S,_,pos(P),_).
elem(N) :- rule(pos(sum(_,S,_)),_), wlist(S,_,neg(N),_).

hold(conjunction(S)) :- eleb(conjunction(S)),
                            hold(P)     : set(S,pos(P)),
                        not hold(N)     : set(S,neg(N)).

hold(sum(L,S,U))     :- eleb(sum(L,S,U)),
                        L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
                                 not hold(N) = W : wlist(S,Q,neg(N),W) ] U.

hold(atom(A))        :- rule(pos(atom(A)),   pos(B)), hold(B).

 L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
          not hold(N) = W : wlist(S,Q,neg(N),W) ] U
                     :- rule(pos(sum(L,S,U)),pos(B)), hold(B).

                     :- rule(pos(false),     pos(B)), hold(B).

#hide. #show hold(atom(A)).
```

Basic meta-encoding
meta.lp

```
litb(B) :- rule(_,B).
litb(E) :- litb(pos(conjunction(S))), set(S,E).
litb(E) :- eleb(sum(_,S,_)), wlist(S,_,E,_).

eleb(P) :- litb(pos(P)).
eleb(N) :- litb(neg(N)).

elem(E) :- eleb(E).
elem(E) :- rule(pos(E),_).
elem(P) :- rule(pos(sum(_,S,_)),_), wlist(S,_,pos(P),_).
elem(N) :- rule(pos(sum(_,S,_)),_), wlist(S,_,neg(N),_).

hold(conjunction(S)) :- eleb(conjunction(S)),
                            hold(P)     : set(S,pos(P)),
                        not hold(N)     : set(S,neg(N)).

hold(sum(L,S,U))     :- eleb(sum(L,S,U)),
                        L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
                                 not hold(N) = W : wlist(S,Q,neg(N),W) ] U.

hold(atom(A))        :- rule(pos(atom(A)),   pos(B)), hold(B).

 L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
          not hold(N) = W : wlist(S,Q,neg(N),W) ] U
                     :- rule(pos(sum(L,S,U)),pos(B)), hold(B).

                     :- rule(pos(false),     pos(B)), hold(B).

#hide. #show hold(atom(A)).
```

Basic meta-encoding meta.lp

```
litb(B) :- rule(_,B).
litb(E) :- litb(pos(conjunction(S))), set(S,E).
litb(E) :- eleb(sum(_,S,_)), wlist(S,_,E,_).

eleb(P) :- litb(pos(P)).
eleb(N) :- litb(neg(N)).

elem(E) :- eleb(E).
elem(E) :- rule(pos(E),_).
elem(P) :- rule(pos(sum(_,S,_)),_), wlist(S,_,pos(P),_).
elem(N) :- rule(pos(sum(_,S,_)),_), wlist(S,_,neg(N),_).

hold(conjunction(S)) :- eleb(conjunction(S)),
                            hold(P)     : set(S,pos(P)),
                        not hold(N)     : set(S,neg(N)).

hold(sum(L,S,U))     :- eleb(sum(L,S,U)),
                        L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
                                 not hold(N) = W : wlist(S,Q,neg(N),W) ] U.

hold(atom(A))        :- rule(pos(atom(A)),   pos(B)), hold(B).

 L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
          not hold(N) = W : wlist(S,Q,neg(N),W) ] U
                     :- rule(pos(sum(L,S,U)),pos(B)), hold(B).

                     :- rule(pos(false),     pos(B)), hold(B).

#hide. #show hold(atom(A)).
```

Basic meta-encoding meta.lp

```
litb(B) :- rule(_,B).
litb(E) :- litb(pos(conjunction(S))), set(S,E).
litb(E) :- eleb(sum(_,S,_)), wlist(S,_,E,_).

eleb(P) :- litb(pos(P)).
eleb(N) :- litb(neg(N)).

elem(E) :- eleb(E).
elem(E) :- rule(pos(E),_).
elem(P) :- rule(pos(sum(_,S,_)),_), wlist(S,_,pos(P),_).
elem(N) :- rule(pos(sum(_,S,_)),_), wlist(S,_,neg(N),_).

hold(conjunction(S)) :- eleb(conjunction(S)),
                            hold(P)     : set(S,pos(P)),
                        not hold(N)     : set(S,neg(N)).

hold(sum(L,S,U))     :- eleb(sum(L,S,U)),
                        L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
                                 not hold(N) = W : wlist(S,Q,neg(N),W) ] U.

hold(atom(A))        :- rule(pos(atom(A)),   pos(B)), hold(B).

 L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
          not hold(N) = W : wlist(S,Q,neg(N),W) ] U
                     :- rule(pos(sum(L,S,U)),pos(B)), hold(B).

                     :- rule(pos(false),     pos(B)), hold(B).

#hide. #show hold(atom(A)).
```

Basic meta-encoding meta.lp

```
litb(B) :- rule(_,B).
litb(E) :- litb(pos(conjunction(S))), set(S,E).
litb(E) :- eleb(sum(_,S,_)), wlist(S,_,E,_).

eleb(P) :- litb(pos(P)).
eleb(N) :- litb(neg(N)).

elem(E) :- eleb(E).
elem(E) :- rule(pos(E),_).
elem(P) :- rule(pos(sum(_,S,_)),_), wlist(S,_,pos(P),_).
elem(N) :- rule(pos(sum(_,S,_)),_), wlist(S,_,neg(N),_).

hold(conjunction(S)) :- eleb(conjunction(S)),
                            hold(P)     : set(S,pos(P)),
                        not hold(N)     : set(S,neg(N)).

hold(sum(L,S,U))     :- eleb(sum(L,S,U)),
                        L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
                                 not hold(N) = W : wlist(S,Q,neg(N),W) ] U.

hold(atom(A))        :- rule(pos(atom(A)),   pos(B)), hold(B).

 L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
          not hold(N) = W : wlist(S,Q,neg(N),W) ] U
                     :- rule(pos(sum(L,S,U)),pos(B)), hold(B).

                     :- rule(pos(false),     pos(B)), hold(B).

#hide. #show hold(atom(A)).
```

Basic meta-encoding meta.lp

```
litb(B) :- rule(_,B).
litb(E) :- litb(pos(conjunction(S))), set(S,E).
litb(E) :- eleb(sum(_,S,_)), wlist(S,_,E,_).

eleb(P) :- litb(pos(P)).
eleb(N) :- litb(neg(N)).

elem(E) :- eleb(E).
elem(E) :- rule(pos(E),_).
elem(P) :- rule(pos(sum(_,S,_)),_), wlist(S,_,pos(P),_).
elem(N) :- rule(pos(sum(_,S,_)),_), wlist(S,_,neg(N),_).

hold(conjunction(S)) :- eleb(conjunction(S)),
                            hold(P)     : set(S,pos(P)),
                        not hold(N)     : set(S,neg(N)).

hold(sum(L,S,U))     :- eleb(sum(L,S,U)),
                        L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
                                 not hold(N) = W : wlist(S,Q,neg(N),W) ] U.

hold(atom(A))        :- rule(pos(atom(A)),   pos(B)), hold(B).

 L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
          not hold(N) = W : wlist(S,Q,neg(N),W) ] U
                     :- rule(pos(sum(L,S,U)),pos(B)), hold(B).

                     :- rule(pos(false),     pos(B)), hold(B).

#hide. #show hold(atom(A)).
```

Basic meta-encoding meta.lp

```
litb(B) :- rule(_,B).
litb(E) :- litb(pos(conjunction(S))), set(S,E).
litb(E) :- eleb(sum(_,S,_)), wlist(S,_,E,_).

eleb(P) :- litb(pos(P)).
eleb(N) :- litb(neg(N)).

elem(E) :- eleb(E).
elem(E) :- rule(pos(E),_).
elem(P) :- rule(pos(sum(_,S,_)),_), wlist(S,_,pos(P),_).
elem(N) :- rule(pos(sum(_,S,_)),_), wlist(S,_,neg(N),_).

hold(conjunction(S)) :- eleb(conjunction(S)),
                            hold(P)     : set(S,pos(P)),
                        not hold(N)     : set(S,neg(N)).

hold(sum(L,S,U))     :- eleb(sum(L,S,U)),
                        L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
                                 not hold(N) = W : wlist(S,Q,neg(N),W) ] U.

hold(atom(A))        :- rule(pos(atom(A)),   pos(B)), hold(B).

 L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
          not hold(N) = W : wlist(S,Q,neg(N),W) ] U
                     :- rule(pos(sum(L,S,U)),pos(B)), hold(B).

                     :- rule(pos(false),     pos(B)), hold(B).

#hide. #show hold(atom(A)).
```

Basic meta-encoding meta.lp

```
litb(B) :- rule(_,B).
litb(E) :- litb(pos(conjunction(S))), set(S,E).
litb(E) :- eleb(sum(_,S,_)), wlist(S,_,E,_).

eleb(P) :- litb(pos(P)).
eleb(N) :- litb(neg(N)).

elem(E) :- eleb(E).
elem(E) :- rule(pos(E),_).
elem(P) :- rule(pos(sum(_,S,_)),_), wlist(S,_,pos(P),_).
elem(N) :- rule(pos(sum(_,S,_)),_), wlist(S,_,neg(N),_).

hold(conjunction(S)) :- eleb(conjunction(S)),
                            hold(P)     : set(S,pos(P)),
                        not hold(N)     : set(S,neg(N)).

hold(sum(L,S,U))     :- eleb(sum(L,S,U)),
                        L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
                                 not hold(N) = W : wlist(S,Q,neg(N),W) ] U.

hold(atom(A))        :- rule(pos(atom(A)),   pos(B)), hold(B).

 L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
          not hold(N) = W : wlist(S,Q,neg(N),W) ] U
                     :- rule(pos(sum(L,S,U)),pos(B)), hold(B).

                     :- rule(pos(false),     pos(B)), hold(B).

#hide. #show hold(atom(A)).
```

Basic meta-encoding meta.lp

```
litb(B) :- rule(_,B).
litb(E) :- litb(pos(conjunction(S))), set(S,E).
litb(E) :- eleb(sum(_,S,_)), wlist(S,_,E,_).

eleb(P) :- litb(pos(P)).
eleb(N) :- litb(neg(N)).

elem(E) :- eleb(E).
elem(E) :- rule(pos(E),_).
elem(P) :- rule(pos(sum(_,S,_)),_), wlist(S,_,pos(P),_).
elem(N) :- rule(pos(sum(_,S,_)),_), wlist(S,_,neg(N),_).

hold(conjunction(S)) :- eleb(conjunction(S)),
                            hold(P)     : set(S,pos(P)),
                        not hold(N)     : set(S,neg(N)).

hold(sum(L,S,U))     :- eleb(sum(L,S,U)),
                        L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
                                 not hold(N) = W : wlist(S,Q,neg(N),W) ] U.

hold(atom(A))        :- rule(pos(atom(A)),   pos(B)), hold(B).

 L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
          not hold(N) = W : wlist(S,Q,neg(N),W) ] U
                     :- rule(pos(sum(L,S,U)),pos(B)), hold(B).

                     :- rule(pos(false),     pos(B)), hold(B).

#hide. #show hold(atom(A)).
```

Basic meta-encoding meta.lp

```
litb(B) :- rule(_,B).
litb(E) :- litb(pos(conjunction(S))), set(S,E).
litb(E) :- eleb(sum(_,S,_)), wlist(S,_,E,_).

eleb(P) :- litb(pos(P)).
eleb(N) :- litb(neg(N)).

elem(E) :- eleb(E).
elem(E) :- rule(pos(E),_).
elem(P) :- rule(pos(sum(_,S,_)),_), wlist(S,_,pos(P),_).
elem(N) :- rule(pos(sum(_,S,_)),_), wlist(S,_,neg(N),_).

hold(conjunction(S)) :- eleb(conjunction(S)),
                            hold(P)     : set(S,pos(P)),
                        not hold(N)     : set(S,neg(N)).

hold(sum(L,S,U))     :- eleb(sum(L,S,U)),
                        L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
                                 not hold(N) = W : wlist(S,Q,neg(N),W) ] U.

hold(atom(A))        :- rule(pos(atom(A)),   pos(B)), hold(B).

 L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
          not hold(N) = W : wlist(S,Q,neg(N),W) ] U
                     :- rule(pos(sum(L,S,U)),pos(B)), hold(B).

                     :- rule(pos(false),     pos(B)), hold(B).

#hide. #show hold(atom(A)).
```

Basic meta-encoding
meta.lp

```
litb(B) :- rule(_,B).
litb(E) :- litb(pos(conjunction(S))), set(S,E).
litb(E) :- eleb(sum(_,S,_)), wlist(S,_,E,_).

eleb(P) :- litb(pos(P)).
eleb(N) :- litb(neg(N)).

elem(E) :- eleb(E).
elem(E) :- rule(pos(E),_).
elem(P) :- rule(pos(sum(_,S,_)),_), wlist(S,_,pos(P),_).
elem(N) :- rule(pos(sum(_,S,_)),_), wlist(S,_,neg(N),_).

hold(conjunction(S)) :- eleb(conjunction(S)),
                            hold(P)     : set(S,pos(P)),
                        not hold(N)     : set(S,neg(N)).

hold(sum(L,S,U))     :- eleb(sum(L,S,U)),
                        L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
                                 not hold(N) = W : wlist(S,Q,neg(N),W) ] U.

hold(atom(A))        :- rule(pos(atom(A)),   pos(B)), hold(B).

 L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
          not hold(N) = W : wlist(S,Q,neg(N),W) ] U
                     :- rule(pos(sum(L,S,U)),pos(B)), hold(B).

                     :- rule(pos(false),     pos(B)), hold(B).

#hide. #show hold(atom(A)).
```

Basic meta-encoding
meta.lp

```
litb(B) :- rule(_,B).
litb(E) :- litb(pos(conjunction(S))), set(S,E).
litb(E) :- eleb(sum(_,S,_)), wlist(S,_,E,_).

eleb(P) :- litb(pos(P)).
eleb(N) :- litb(neg(N)).

elem(E) :- eleb(E).
elem(E) :- rule(pos(E),_).
elem(P) :- rule(pos(sum(_,S,_)),_), wlist(S,_,pos(P),_).
elem(N) :- rule(pos(sum(_,S,_)),_), wlist(S,_,neg(N),_).

hold(conjunction(S)) :- eleb(conjunction(S)),
                            hold(P)     : set(S,pos(P)),
                        not hold(N)     : set(S,neg(N)).

hold(sum(L,S,U))     :- eleb(sum(L,S,U)),
                        L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
                                 not hold(N) = W : wlist(S,Q,neg(N),W) ] U.

hold(atom(A))        :- rule(pos(atom(A)),   pos(B)), hold(B).

 L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
          not hold(N) = W : wlist(S,Q,neg(N),W) ] U
                     :- rule(pos(sum(L,S,U)),pos(B)), hold(B).

                     :- rule(pos(false),     pos(B)), hold(B).

#hide. #show hold(atom(A)).
```

Basic meta-encoding meta.lp

```
litb(B) :- rule(_,B).
litb(E) :- litb(pos(conjunction(S))), set(S,E).
litb(E) :- eleb(sum(_,S,_)), wlist(S,_,E,_).

eleb(P) :- litb(pos(P)).
eleb(N) :- litb(neg(N)).

elem(E) :- eleb(E).
elem(E) :- rule(pos(E),_).
elem(P) :- rule(pos(sum(_,S,_)),_), wlist(S,_,pos(P),_).
elem(N) :- rule(pos(sum(_,S,_)),_), wlist(S,_,neg(N),_).

hold(conjunction(S)) :- eleb(conjunction(S)),
                            hold(P)     : set(S,pos(P)),
                        not hold(N)     : set(S,neg(N)).

hold(sum(L,S,U))     :- eleb(sum(L,S,U)),
                        L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
                                 not hold(N) = W : wlist(S,Q,neg(N),W) ] U.

hold(atom(A))        :- rule(pos(atom(A)),   pos(B)), hold(B).

 L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
          not hold(N) = W : wlist(S,Q,neg(N),W) ] U
                     :- rule(pos(sum(L,S,U)),pos(B)), hold(B).

                     :- rule(pos(false),     pos(B)), hold(B).

#hide. #show hold(atom(A)).
```

Basic meta-encoding meta.lp

```
litb(B) :- rule(_,B).
litb(E) :- litb(pos(conjunction(S))), set(S,E).
litb(E) :- eleb(sum(_,S,_)), wlist(S,_,E,_).

eleb(P) :- litb(pos(P)).
eleb(N) :- litb(neg(N)).

elem(E) :- eleb(E).
elem(E) :- rule(pos(E),_).
elem(P) :- rule(pos(sum(_,S,_)),_), wlist(S,_,pos(P),_).
elem(N) :- rule(pos(sum(_,S,_)),_), wlist(S,_,neg(N),_).

hold(conjunction(S)) :- eleb(conjunction(S)),
                            hold(P)     : set(S,pos(P)),
                        not hold(N)     : set(S,neg(N)).

hold(sum(L,S,U))     :- eleb(sum(L,S,U)),
                        L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
                                 not hold(N) = W : wlist(S,Q,neg(N),W) ] U.

hold(atom(A))        :- rule(pos(atom(A)),   pos(B)), hold(B).

 L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
          not hold(N) = W : wlist(S,Q,neg(N),W) ] U
                     :- rule(pos(sum(L,S,U)),pos(B)), hold(B).

                     :- rule(pos(false),     pos(B)), hold(B).

#hide. #show hold(atom(A)).
```

Basic meta-encoding
meta.lp

```
litb(B) :- rule(_,B).
litb(E) :- litb(pos(conjunction(S))), set(S,E).
litb(E) :- eleb(sum(_,S,_)), wlist(S,_,E,_).

eleb(P) :- litb(pos(P)).
eleb(N) :- litb(neg(N)).

elem(E) :- eleb(E).
elem(E) :- rule(pos(E),_).
elem(P) :- rule(pos(sum(_,S,_)),_), wlist(S,_,pos(P),_).
elem(N) :- rule(pos(sum(_,S,_)),_), wlist(S,_,neg(N),_).

hold(conjunction(S)) :- eleb(conjunction(S)),
                            hold(P)     : set(S,pos(P)),
                        not hold(N)     : set(S,neg(N)).

hold(sum(L,S,U))     :- eleb(sum(L,S,U)),
                        L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
                                 not hold(N) = W : wlist(S,Q,neg(N),W) ] U.

hold(atom(A))        :- rule(pos(atom(A)),   pos(B)), hold(B).

 L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
          not hold(N) = W : wlist(S,Q,neg(N),W) ] U
                     :- rule(pos(sum(L,S,U)),pos(B)), hold(B).

                     :- rule(pos(false),     pos(B)), hold(B).

#hide. #show hold(atom(A)).
```

Basic meta-encoding
meta.lp

```
litb(B) :- rule(_,B).
litb(E) :- litb(pos(conjunction(S))), set(S,E).
litb(E) :- eleb(sum(_,S,_)), wlist(S,_,E,_).

eleb(P) :- litb(pos(P)).
eleb(N) :- litb(neg(N)).

elem(E) :- eleb(E).
elem(E) :- rule(pos(E),_).
elem(P) :- rule(pos(sum(_,S,_)),_), wlist(S,_,pos(P),_).
elem(N) :- rule(pos(sum(_,S,_)),_), wlist(S,_,neg(N),_).

hold(conjunction(S)) :- eleb(conjunction(S)),
                            hold(P)     : set(S,pos(P)),
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hold(sum(L,S,U))     :- eleb(sum(L,S,U)),
                        L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
                                 not hold(N) = W : wlist(S,Q,neg(N),W) ] U.

hold(atom(A))        :- rule(pos(atom(A)),   pos(B)), hold(B).

 L #sum [     hold(P) = W : wlist(S,Q,pos(P),W),
          not hold(N) = W : wlist(S,Q,neg(N),W) ] U
                     :- rule(pos(sum(L,S,U)),pos(B)), hold(B).

                     :- rule(pos(false),     pos(B)), hold(B).

#hide. #show hold(atom(A)).
```

Basic meta-encoding
`meta.lp`

```
litb(B) :- rule(_,B).
litb(E) :- litb(pos(conjunction(S))), set(S,E).
litb(E) :- eleb(sum(_,S,_)), wlist(S,_,E,_).

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                     :- rule(pos(false),     pos(B)), hold(B).

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                            hold(P)     : set(S,pos(P)),
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meta.lp

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                     :- rule(pos(sum(L,S,U)),pos(B)), hold(B).

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meta.lp

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                     :- rule(pos(sum(L,S,U)),pos(B)), hold(B).

                     :- rule(pos(false),     pos(B)), hold(B).

#hide. #show hold(atom(A)).
```

Reified Grounding by example

```
gringo --reify easy.lp | gringo - meta.lp
```

```
eleb(atom(p(1))).       litb(pos(atom(p(1)))).       elem(atom(p(1))).       elem(false).
eleb(atom(p(2))).       litb(pos(atom(p(2)))).       elem(atom(p(2))).       elem(sum(0,0,2)).
eleb(atom(p(3))).       litb(pos(atom(p(3)))).       elem(atom(p(3))).       elem(sum(0,0,3)).
eleb(conjunction(0)).   litb(pos(conjunction(0))).   elem(atom(q(2))).
eleb(conjunction(1)).   litb(pos(conjunction(1))).   elem(conjunction(0)).
eleb(conjunction(2)).   litb(pos(conjunction(2))).   elem(conjunction(1)).
eleb(sum(0,0,2)).       litb(pos(sum(0,0,2))).       elem(conjunction(2)).

wlist(0,0,pos(atom(p(1))),1).
wlist(0,1,pos(atom(p(2))),1).
wlist(0,2,pos(atom(p(3))),1).
rule(pos(sum(0,0,3)),pos(conjunction(0))).
set(1,pos(sum(0,0,2))).
rule(pos(false),pos(conjunction(1))).
set(2,pos(atom(p(2)))).
set(2,pos(atom(p(3)))).
rule(pos(atom(q(2))),pos(conjunction(2))).

hold(conjunction(2)) :- hold(atom(p(3))),hold(atom(p(2))).
hold(conjunction(1)) :- hold(sum(0,0,2)).
hold(conjunction(0)).
hold(sum(0,0,2)) :- 0 #sum [ hold(atom(p(3)))=1, hold(atom(p(2)))=1, hold(atom(p(1)))=1 ] 2.
hold(atom(q(2))) :- hold(conjunction(2)).
0 #sum [ hold(atom(p(3)))=1, hold(atom(p(2)))=1, hold(atom(p(1)))=1 ] 3.
 :- hold(conjunction(1)).

#hide.
```

Reified Grounding by example

```
gringo --reify easy.lp | gringo - meta.lp
```

```
eleb(atom(p(1))).       litb(pos(atom(p(1)))).       elem(atom(p(1))).       elem(false).
eleb(atom(p(2))).       litb(pos(atom(p(2)))).       elem(atom(p(2))).       elem(sum(0,0,2)).
eleb(atom(p(3))).       litb(pos(atom(p(3)))).       elem(atom(p(3))).       elem(sum(0,0,3)).
eleb(conjunction(0)).   litb(pos(conjunction(0))).   elem(atom(q(2))).
eleb(conjunction(1)).   litb(pos(conjunction(1))).   elem(conjunction(0)).
eleb(conjunction(2)).   litb(pos(conjunction(2))).   elem(conjunction(1)).
eleb(sum(0,0,2)).       litb(pos(sum(0,0,2))).       elem(conjunction(2)).

wlist(0,0,pos(atom(p(1))),1).
wlist(0,1,pos(atom(p(2))),1).
wlist(0,2,pos(atom(p(3))),1).
rule(pos(sum(0,0,3)),pos(conjunction(0))).
set(1,pos(sum(0,0,2))).
rule(pos(false),pos(conjunction(1))).
set(2,pos(atom(p(2)))).
set(2,pos(atom(p(3)))).
rule(pos(atom(q(2))),pos(conjunction(2))).

hold(conjunction(2)) :- hold(atom(p(3))),hold(atom(p(2))).
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0 #sum [ hold(atom(p(3)))=1, hold(atom(p(2)))=1, hold(atom(p(1)))=1 ] 3.
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Reified Grounding by example

```
gringo --reify easy.lp | gringo - meta.lp
```

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eleb(atom(p(1))).       litb(pos(atom(p(1)))).       elem(atom(p(1))).       elem(false).
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eleb(atom(p(3))).       litb(pos(atom(p(3)))).       elem(atom(p(3))).       elem(sum(0,0,3)).
eleb(conjunction(0)).   litb(pos(conjunction(0))).   elem(atom(q(2))).
eleb(conjunction(1)).   litb(pos(conjunction(1))).   elem(conjunction(0)).
eleb(conjunction(2)).   litb(pos(conjunction(2))).   elem(conjunction(1)).
eleb(sum(0,0,2)).       litb(pos(sum(0,0,2))).       elem(conjunction(2)).

wlist(0,0,pos(atom(p(1))),1).
wlist(0,1,pos(atom(p(2))),1).
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rule(pos(sum(0,0,3)),pos(conjunction(0))).
set(1,pos(sum(0,0,2))).
rule(pos(false),pos(conjunction(1))).
set(2,pos(atom(p(2)))).
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hold(conjunction(2)) :- hold(atom(p(3))),hold(atom(p(2))).
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#hide.
```

Reified Grounding by example

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```

```
eleb(atom(p(1))).       litb(pos(atom(p(1)))).       elem(atom(p(1))).       elem(false).
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eleb(conjunction(2)).   litb(pos(conjunction(2))).   elem(conjunction(1)).
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#hide.
```

Solving by example

easy.lp

```
{ p(1..3) }.
:- { p(X) } 2.
q(X) :- p(X), p(X+1), X>1.
```

```
gringo --reify easy.lp | gringo - meta.lp | clasp 0
```

```
clasp version 2.0.0
Reading from stdin
Solving...
Answer: 1
hold(atom(p(3))) hold(atom(p(2))) hold(atom(p(1))) hold(atom(q(2)))
SATISFIABLE

Models      : 1
Time        : 0.000s
CPU Time    : 0.000s
```

Solving by example

easy.lp

```
{ p(1..3) }.
:- { p(X) } 2.
q(X) :- p(X), p(X+1), X>1.
```

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clasp version 2.0.0
Reading from stdin
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Answer: 1
hold(atom(p(3))) hold(atom(p(2))) hold(atom(p(1))) hold(atom(q(2)))
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Models      : 1
Time        : 0.000s
CPU Time    : 0.000s
```
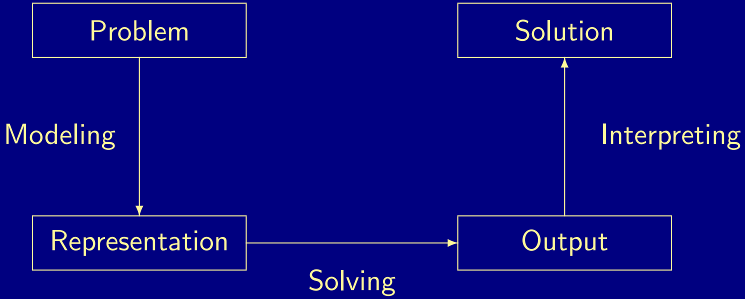

Outline

Declarative problem solving

- *"What is the problem?"*

  instead of
- *"How to solve the problem?"*



Declarative problem solving

- *“What is the problem?”*
  instead of
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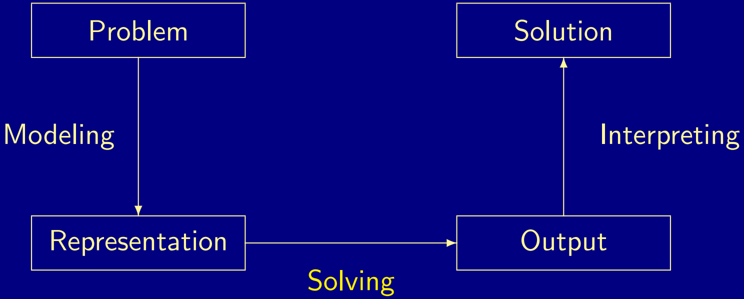


Towards conflict-driven search

Boolean constraint solving algorithms pioneered for SAT led to:

- Traditional DPLL-style approach
  (DPLL stands for 'Davis-Putnam-Logemann-Loveland')
  - (Unit) propagation
  - (Chronological) backtracking
  - in ASP, eg *smodels*
- Modern CDCL-style approach
  (CDCL stands for 'Conflict-Driven Constraint Learning')
  - (Unit) propagation
  - Conflict analysis (via resolution)
  - Learning + Backjumping + Assertion
  - in ASP, eg *clasp*

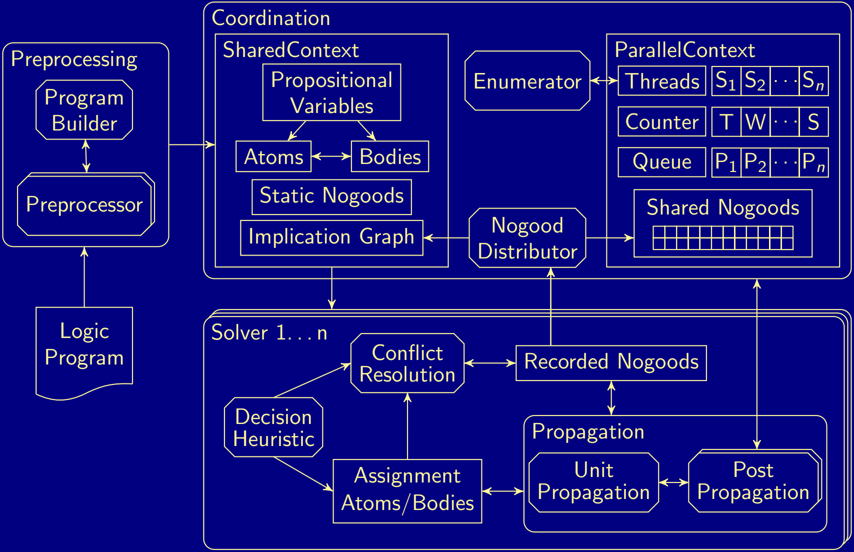
DPLL-style solving

```
loop
    propagate                                  // deterministically assign literals
    if no conflict then
        if all variables assigned then return solution
        else decide                            // non-deterministically assign some literal
    else
        if top-level conflict then return unsatisfiable
        else
            backtrack                          // unassign literals made after last decision
            flip                               // assign complement of last decision literal
```

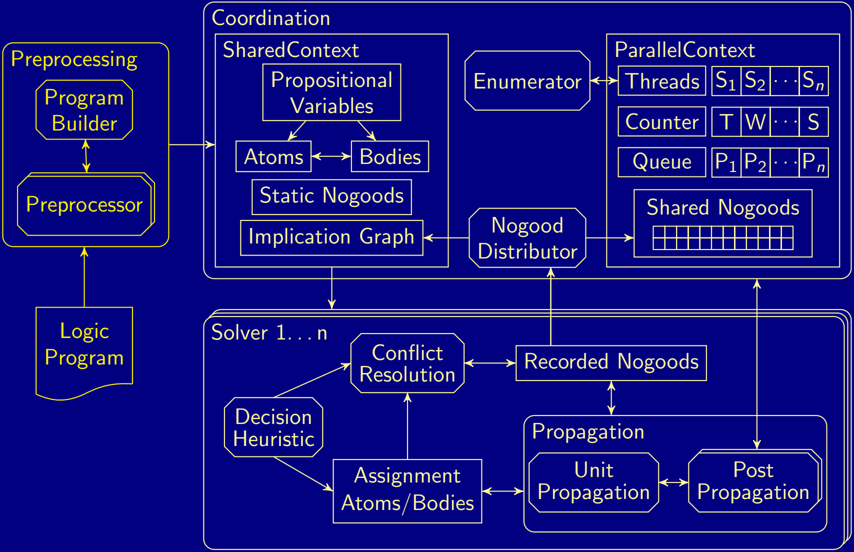
CDCL-style solving

```
loop
    propagate                                  // deterministically assign literals
    if no conflict then
        if all variables assigned then return solution
        else decide                            // non-deterministically assign some literal
    else
        if top-level conflict then return unsatisfiable
        else
            analyze                            // analyze conflict and add conflict constraint
            backjump                           // unassign literals until conflict constraint is unit
```

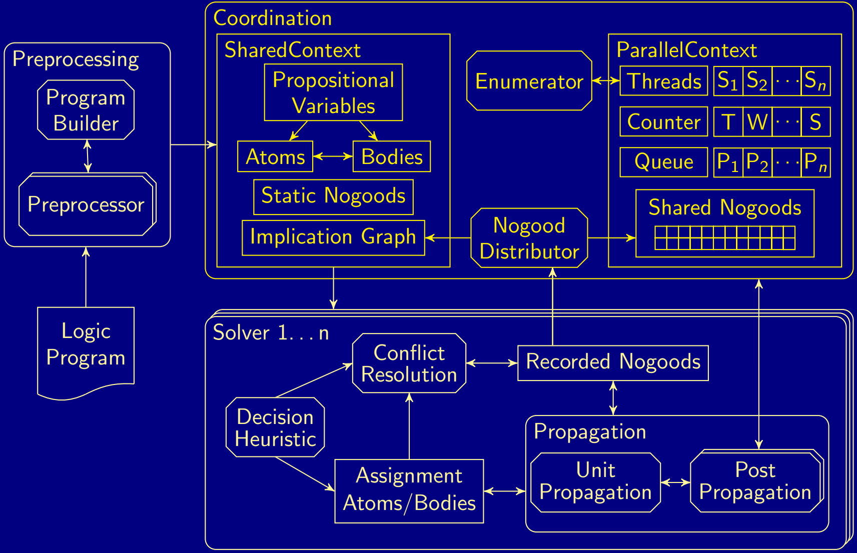
Multi-threaded architecture of *clasp*



Multi-threaded architecture of *clasp*



Multi-threaded architecture of *clasp*



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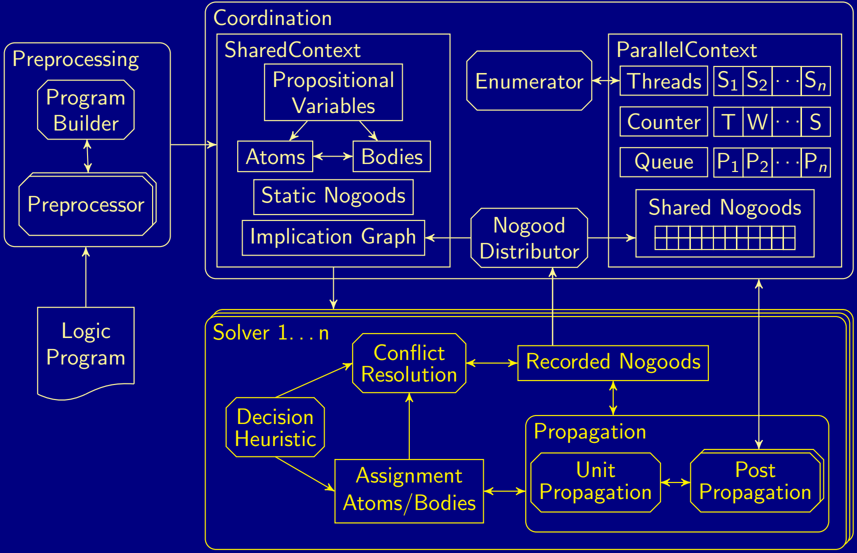

Outline

`http://potassco.sourceforge.net`

Potassco, the Potsdam Answer Set Solving Collection,
bundles tools for ASP developed at the University of Potsdam,
for instance:

- *Grounder*: gringo, pyngo
- *Solver*: clasp, {a,u,h}clasp, claspD, claspfolio, claspar, aspeed
- *Grounder+Solver*: Clingo, iClingo, oClingo, Clingcon
- *Further Tools*: as{un}cud, coala, fimo, metasp, plasp, etc.

Benchmarking: `http://asparagus.cs.uni-potsdam.de`

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Outline

Summary

- ASP is emerging as a viable tool for Knowledge Representation and Reasoning
- ASP offers efficient and versatile off-the-shelf solving technology
  - `http://potassco.sourceforge.net`
  - ASP, CASC, MISC, PB, and SAT competitions
- ASP offers an expanding functionality and ease of use
  - Rapid application development tool
- ASP has a growing range of applications

Summary

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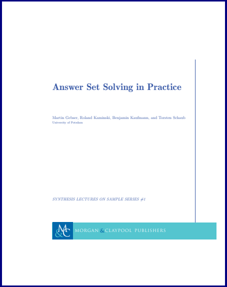
ASP = DB+LP+KR+SAT

The (forthcoming) Potassco Book

1. Motivation
2. Introduction
3. Basic modeling
4. Grounding
5. Characterizations
6. Solving
7. Systems
8. Advanced modeling
9. Conclusions



Answer Set Solving in Practice

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